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THE ANALYSIS OF COLLEGE STUDENTS' DIFFICULTIES IN PRODUCING PROOFS

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Abstract

Learning difficultness is a condition which the student can't learn as usual because of the threat and disturbances in learning. Learning difficultness faced by students happened when they followed the lessons from the teacher. Based on result observation of real analysis course work known that many students got the difficultness when prove the proposition given by the teacher. The research results showed that the students got contents understanding difficulties, such as misunderstanding of definitions, not enough word and lack of understanding. It means that they got difficulties with mathematics processes. There are many student identified proving using examples to be invalid. They use specific examples to prove general statements.

Keywords: learning difficultness,mathematical proof, students error

INTRODUCTION

Mathematics is as such an essential discipline because of the important role it plays in the individual's personal life. It is considered as symbolic and universal language that enables human beings to thinks, record and communicates ideas concerning the elements and relationship of quantity. Mathematical skills are abstract skills that involve mapping of language into symbols. The basic purpose of learning mathematics should be problem solving, which involves presenting a problem and developing the skills needed to solve that problem. Every individual is required to make effective use of the quantitative information available in his/her environment in problems of day-to-day life (Jena, 2013: 68). Luce and Suppes in Jena (2013) suggest that future mathematical research should address to the content of mathematics, concentrating particularly on the character of mathematical thinking rather than on nature of mathematical objects.

Yin Ko and Eric (2009) say that proving and refuting are crucial abilities in advanced mathematical thinking because they help demonstrate whether and why propositions are true or false. In the mathematics community, proving and refuting are inextricably linked given the role each plays in establishing mathematical knowledge. A mathematical proof requires that definitions, statements or procedures are used to deduce the truth of one statement from another, helping people understand the logic behind a statement and insight into how and why it works. Mathematical proofs can provide students with insight into meanings behind statements and also help them see why statements are true or false. Accordingly, undergraduate mathematics students are expected to learn and to use proofs throughout the undergraduate mathematics curriculum.

Looking at the processes involved in proving and how students understand proofs are thoroughly addresses topics. Of particular interest to us is how students verify and validate proofs and the misconceptions in proving. Stylianides in Stavrou (2014) reports students' misconception that empirical arguments constitute valid proofs. That is, students use specific examples to prove general statements. Pfeiffer in Stavrou (2014) conducted a study in which she presented mathematical proofs containing errors to first year mathematics undergraduates and asked them to evaluate and criticize the statements. Pfeiffer notes that many students identified proving using examples to be invalid. (Stavrou, 2014: 1-2)

This paper considers undergraduate educations students who are take real analysis course. We examine common errors that these students make when proving statements in their course work.

Our purpose then is to gathering and analyzing these common mistakes. This paper was guided by questions: What are the common errors that mathematics education students make when writing proofs. Accordingly, from this question we know the learning difficultness of mathematics undergraduate students in Muhammadiyah University of Ponorogo.

LITERATURE REVIEW

Learning difficultness is a condition which the student can't learn as usual because of the threat and disturbances in learning. Learning difficultness faced by students happened when followed the lessons from the teachers. Trying to detect learning difficulties can also be done with different perspectives and there are several general questions to consider, for example: Are the measures proper, e.g. examinations and other tests? What are the common informal indications? Do we need special diagnostic tools than the ones that seem more common in primary school? Can students recognize and understand their own difficulties? (Lithner, 2011:290-291).

Learning difficulties in mathematics can lead different meaning. Different things and the problem are not just makes misunderstand each other, but also that different meanings may lead to different conclusions and actions. The characteristics of learning difficulties can be related to educational goals that some students fail to reach to various degrees. These goals can, in a similar way as in for example the NCTM Standards (NCTM, 2000) for K-12 education, be seen as comprising both content and processes to master.

1. Content understanding difficulties
   Many learning and achievement difficulties are directly related to inherent mathematical difficulties within specified concepts. For example, concerning calculus the difficulties found in the research literature seem related to all of its fundamental notions, e.g. variable, function, limit, derivative, integral and differential equation. There are other literature examples related to linear algebra, abstract algebra and real analysis. It is clear that students' concept images often differ substantially from the concept definitions, not only in the sense that they are incomplete but also in characteristics. These difficulties are at least partially explained by theory indicating the complexity of mental concept formations. It is also clear that a conceptual understanding is not sufficient for mathematical proficiency.
2. Difficulties with mathematical processes
   There are findings related to difficulties with what in the NTCM Principles and Standards (NCTM, 2000) are denoted processes, e.g. non-routine problem-solving, proof and proving, reasoning, representing and modeling. There are many studies on different aspects of learning, understanding and implementing proof. Students have difficulties in differing proofs from other less rigorous types of argumentation, understanding proof statements, making the transition from informal to formal reasoning and constructing proofs (Lithner, 2011:291-293).

Stefanonowicz (2014) stated that a proof was a sequence of logical statement, one implying another, which given as explanation of why a statement is true. Previously established theorems may be used to deduce the new ones, one may also refer to axioms, which are the starting points, "rule" accepted by everyone. Mathematical proof is absolute, which means that once a theorem is proved, it is proved forever. Until proven though, the statement is never accepted as a true one. Writing proofs is the essence of mathematics study. Every word will be defined, notations clearly presented and each theorem proved. Students learn how to construct logical arguments and what a good proof looks like. It is not easy though and requires practice, there for it is always tempting for students to learn theorems and apply them, leaving proofs behind.

Yin ko and Eric (2009) say that from a traditional perspective, "a mathematical proof is a formal and logical line of reasoning that begins with a set of axioms and moves through steps to a conclusion". A mathematical proof requires some aspects, such as the essential components of sets of accepted statement, modes of argumentation and modes of argument representation. In this statement, proof serves as a means to communicate thoughts with learners in the

mathematics community. Common sense suggests that individuals who understand what constitutes a mathematical proof may be more successful at evaluating purported arguments or their written responses as a valid proof or not.

In order to better characterize undergraduates' proof productions for a true proposition, three proof classifications applied to the study reported here. The first proof category, the inductive proof scheme, describes how individuals convince themselves or persuade others by providing one or more particular examples, which corresponds to naive empiricism (verification by several randomly selected cases) and crucial experiment (verification by carefully selected cases). The second proof category, non-referential symbolic proof scheme, demonstrates that individuals employ symbolic manipulations with little or no coherent understandings of their meanings. In other words, students manipulate symbols with no "functional or quantitative reference[s]". The third proof category, the structural proof scheme, suggests that individuals realize that "definitions and theorems belong in the structure created by a particular set of axioms". According to Balacheff in Yin Ko (2009), calculations on statements mean students rely on definitions, theorems, or explicit properties related to the statement when producing a proof. Similarly, Weber and Alcock in Yon Ko (2009) described how an individual attempts to construct a proof by stating the mathematical definition or using related facts that he or she knows about the concepts of producing proofs as defined by syntactic proof productions (Yin ko, 2009: 69).

METHOD

This research is a qualitative research. This research was conducted in two classes at Mathematics Department, Teacher Training and Education Faculty, Muhammadiyah University of Ponorogo. The course work of undergraduate students in mathematics department who take real analysis course was observed here. Real analysis course is proof-based and require that students be able to write and understand mathematical proofs.

The data were collected by correcting the student works in doing real numbers system problems. As their assignments were being graded and compiled a list of errors made by the students. From this list, the most common errors were noted and described. The purpose of this is to answer the research question of what the common errors are that education students make when proving.

Instruments validated by two lecturers who are experts in their fields. The instruments are used to identify the student error when proving. Instrument is taken from textbook that is used in real analysis course.

DISCUSSION

In introduction has been stated that the question of this research is "what is the common error that mathematics education students make when writing proofs?" Below are sample examples:

1. Show that, if $a, b \in \mathbb{R}$, then $\min\{a,b\} = \frac{1}{2}(a+b-|a-b|)$.

A sample solution is:

Let $a = 5$ and $b = 7$, then on the left side we get $\min\{a,b\} = \min\{5,7\} = 5$. On the right side, we get $\frac{1}{2}(a+b-|a-b|) = \frac{1}{2}(5+7-|5-7|) = \frac{1}{2}(12-2) = 5$. Therefore, we have $\min\{a,b\} = \frac{1}{2}(a+b-|a-b|)$.

The above example is representative of some student work who was proving general statements using specific examples. A proof is a deductive argument for a mathematical statement. Based on student work above, it seemed that student do not understand that proving processes in mathematics can't be done by specific case. They think if a proposition can be applied in a special case, it leads to general case.

2. If x and y irrational numbers, are $x + y$ irrational?

A sample solution is:

Let $x = \sqrt{2}$ and $y = -\sqrt{2}$ then $x + y = \sqrt{2} - \sqrt{2} = 0$ is irrational. Hence, if x and y irrational, then $x + y$ is irrational.

3. If x and y irrational numbers, are xy irrational number?

A sample solution is:

Let $x = \sqrt{2}$ and $y = \sqrt{2}$ then $xy = \sqrt{2}\sqrt{2} = 2$. Then, if x, y irrational number then xy rational number.

4. If x and y irrational numbers, are x^y irrational?

A sample solution is:

Let $x = \sqrt{3}$ and $y = \sqrt{2}$ then $x^y = \sqrt{3}^{\sqrt{2}}$ irrational.

The example given above is representative of the responses from some students who almost provided an example to make the answer of that question is false. These students seemed have assuming that the conclusion of the statement is true, so they provide an example that supported the truth value of the statement "if x and y irrational, then $x + y, xy$ and x^y irrational." It showed that if the student did not master the topic mathematical proofs well, so the student found the difficult. It seemed they get problem in counter example concept understanding.

5. Use the definition of the limits of a sequence to establish the following limits.

$$\lim\left(\frac{n^2}{2n^2+1}\right) = \frac{1}{2}.$$

A sample solution is:

$$\lim\left(\frac{n^2}{2n^2+1}\right) = \lim\left(\frac{\frac{n^2}{n^2}}{\frac{2n^2}{n^2}+\frac{1}{n^2}}\right) = \lim\left(\frac{1}{2+\frac{1}{n^2}}\right) = \frac{1}{2}$$

Based on the student's work, it showed that student provided an incorrect proof with inadequate understandings of limits of sequences to prove use formal definition. These students did not apply the definitions correctly. In Stavrou (2014) have mentioned that Edwards and Ward have mathematics education research about "Student (mis) use of mathematical definitions", they got results that nothing that students did not understand the role formal definitions play in mathematics. Firstly, students do not understand that definitions are stipulated and context-dependent. Secondly, students that can correctly state definitions cannot necessarily apply them correctly, if at all, "even in the apparent absence of any other course of action".

6. Use the definition of the limits of functions to establish the following limits.

$$\lim_{x\to 0}\frac{x^2}{|x|} = 0$$

A sample solution:

Let $f(x) \coloneqq \frac{x^2}{|x|}$ for all $x \in \mathbb{R}$ and $L = 0$. $\forall\varepsilon > 0, \exists\delta > 0$. If $0 < |x - \delta| < \delta$,

$$|f(x) - L| = \left|\frac{x^2}{|x|} - 0\right| = \frac{x^2}{|x|} = |x| < \varepsilon.$$

Based on student's work above, the students can write down the definition of limit of functions. But, there are some missing steps on proving processes above, such as the determination of δ. Whereas, proving limits of functions case, the essence of proof lies on δ determination. Besides that, students did not justify the proofs, but they use symbols only. It seemed that they are trying to memorize the definition rather than understand them.

CONCLUSIONS

Writing proofs in mathematics course is different from what has been done during school, so it will take some time to get used to do this properly. The students get a type a difficultness that is content understanding difficultness and difficulties with mathematical processes. A common mistake that students make when trying to present a proof, for content understanding difficultness such as:

1. Misunderstanding of definitions.
   Students do not know where to start because they do not know the definitions of the objects they are working with.
2. Not enough word.
   Students don't explain the step in their arguments, as if they think they are not allowed to use word, only symbols.
3. Lack of understanding.
   When a student gets to a point in a proof that they are cannot proceed from, often the conclusion of the result follow immediately and it is clear that the student does not understand the necessary missing arguments.

University mathematics students is not only get context understanding difficultness but also mathematics processes difficultness, such as incorrect steps. For example they prove by using specific example.

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