Tadris: Jurnal Keguruan dan Ilmu Tarbiyah 3 (2): 155-166 (2018)
DOI: 10.24042/tadris.v3i2.3109

TADRIS: JURNAL KEGURUAN DAN ILMU TARBIYAH
(Tadris: Journal of Education and Teacher Training)
P-ISSN: 2301-7562 | E-ISSN: 2579-7964
ejournal.radenintan.ac.id/index.php/tadris/index

Profile of Students' Errors in Mathematical Proof Process Viewed from Adversity Quotient (AQ)

Arta Ekayanti[1]*, Hikma Khilda Nasyiithoh[2]
[1]Faculty of Education and Teacher Training, Universitas Muhammadiyah Ponorogo, Indonesia
[2]Faculty of Education and Teacher Training, Institut Agama Islam Negeri Ponorogo, Indonesia

Article History:
Received: October 19th, 2018
Revised: November 25th, 2018
Accepted: December 20th, 2018
Published: December 28th, 2018

Keywords:
Adversity Quotient,
Newmann Error Analysis,
Students' error.

***Correspondence Address:**
arta_ekayanti@ymail.com

Abstract: Mathematical proof is an important aspect in mathematics, especially in analysis. An error in the mathematical proof construction process often occurs. This study aims to analyze the students' errors in producing proof. Each of the categories of students' Adversity Quotient (AQ) is identified related to the type of students' error. The type of students' errors used according to Newmann's Error Analysis. This study used a qualitative approach. This study was conducted to 25 students who were taking real analysis course. Documentation, test, and interview were used to gather the data. Analyzing the students' test result and then interviewing them for each AQ category were done for the analysis process. The results show that there are 48% climber students, 52% camper students, and no one is identified as a quitter student. Climber students tend to make some proving error such as transformation error, process skill error, and encoding error while camper students make the comprehension error, transformation error, process skill error, and encoding error when they are producing proof.

INTRODUCTION

Analysis is one branch of mathematics. This is stated in one of the courses called real analysis. It should be noted that mathematics is not merely numbers. This is in line with Hernadis' (Hernadi, 2016), an opinion which says that so far the views on mathematics were still within the scope of the calculation activities relating to variables and numbers. However, it should be noted that important activities in the study of mathematics are mathematical proof the truth or facts applied and communicated in mathematics. Therefore, Yi Yin Ko and Eric Knuth say that mathematical proof is one of the basic abilities for advanced mathematical thinking (Ko & Knuth, 2009). Besides that, Knuth also says that mathematical proof plays a role in systematizing statements into axiomatic systems (Sucipto & Mauliddin, 2016). Mathematical proof includes thinking about new concepts, focusing on important aspects, using relevant prior knowledge, defining new things (if needed), and compiling valid argument (Hidayat, 2017; C. K. Sari, Waluyo, Ainur, & Darmaningsih, 2018). This must be based on a deductive mindset so that students are able to understand the mathematical proof process (Ekayanti, 2017). There is often a misunderstanding in solving mathematical proof problems, including the use of empirical arguments in the process of mathematical proof

(Stavrou, 2014). This is not an easy job, and it can be seen from the many errors made by students in completing mathematical proof cases.

Some errors in mathematical proof occur because students have not fully understood the true nature of mathematical proof. Students still often do mathematical proof using examples. Of course, this is invalid for the mathematical proof process. Besides that, the argument used is illogical. There are times when the mathematical proof made does not conclude. This can occur because of failures or errors in the first few stages are failing to reach the next stage (D. P. Sari, Darhim, & Rosjanuardi, 2018; Wijaya, Heuvel-panhuizen, Doorman, & Robitzch, 2014). The problem, critical thinking skills are needed so that students can plan and execute it effectively and accurately (Sukoriyanto, Nusantara, Subanji, & Chandra, 2016). This also applies in the case of mathematical proof where the students are required to have tenacity and resilience in facing existing difficulties.

Tenacity and resilience in facing challenges or difficulties are called Adversity Quotient (Stoltz, 2000). Stoltz divides three types of AQ, namely quitters, campers and climbers. The quitters tend to lack the willingness to accept the challenges that exist in their lives. The campers already have the willingness to try facing the challenges and problems, but this type of individual thinks that the effort is enough. The climbers tend to survive and struggle in facing problems, challenges, and obstacles (Yanti, Koestoro, & Sutiarso, 2018).

Considering that, the real analysis course is more dominated by mathematical proof, including in the rules of proof derived from formal definitions, as well as the theorems or lemmas associated previously (Ah, 2016). This is considered a difficult thing for students. Because of these difficulties, AQ is needed in learning mathematics (Guntur Suhandoyo, 2016). Therefore, this research was carried out in real analysis course to know more about the types of errors made by the students in learning mathematics, especially mathematical proof in terms of Adversity Quotient.

THEORETICAL SUPPORT

Hernadi says that mathematical proof is a method of communicating a mathematical truth to others who also understand the language of mathematics (Hernadi, 2016). A proof is a series of logical arguments that explain the truth of a statement or proposition. (Stefanowich, 2014) states that proof is a series of logical statements, where one statement influences the other statement, of course, there must be a valid explanation of the truth of the statement. Logically, in this case, it is intended that each step in the mathematical proof must be based on previous steps or other facts with guaranteed truth.

Anne Newmann classified types of errors into five types, including reading errors, comprehension errors, transformation errors, process skill errors, and encoding errors (Bagus Nur Iman, Toto Nusantara, 2016). Students are said to make a reading error if they experience errors in reading and understanding the command of the questions and errors in recognizing the symbols on the question. Comprehension error occurs when the students did not know what is known and asked from the question. Transformation errors occur if students experience errors in determining problem-solving strategies. Students experience a process skill error if they make algebraic operational errors and are wrong in carrying out completion procedures. While encoding errors occur when the students are able to determine the solution to the problem, but they are unable to write the procedure and form the answer correctly.

Intelligence is one of the psychological factors that influence learning achievement (Leonard, 2017). There are several types of intelligence including Adversity Quotient. Adversity Quotient (AQ) is a person's ability to struggle with and overcome obstacles, difficulties, or problems that exist and will turn them into opportunities for success (Stoltz, 2000). Understanding the importance of AQ in achieving success will encourage the students to always struggle in the learning process even though they must face various obstacles and difficulties (Rukmana & Paloloang, 2016). AQ possessed by each individual in facing and overcoming difficulties is different. The level of ability possessed will have an impact on the ability to go through life and be able to provide great benefits for success (Nurhayati, 2015). Stoltz illustrates that life is like climbing a mountain. Therefore, Stoltz divides AQ into three types, namely Quitters (groups of individuals who stop) are groups of individuals who lack the willingness to accept the challenges that exist in their lives. The quitter will be more likely to reject challenges or problems (Hidayat, Herdiman, Aripin, Yuliani, & Maya, 2018; Christina Kartika Sari, Sutopo, & Aryuna, 2016). In the world of education, students who belong to the quitter type are students who are easy to give up and despair in facing the problems. Campers (groups of individuals who camp) are groups of individuals who already have the will to try to deal with challenges and problems but then they feel that it is enough. These individual groups prefer safe situations or prefer to be in a comfort zone. Students who belong to the campers usually type already struggle, but one factor could make them give up and eventually lost the challenge. Climbers are groups of individuals who tend to survive and struggle in facing problems, challenges, and obstacles. Students who belong to the climber type are learners who always sought and unyielding (Wardiana, 2014; Yani, Ikhsan, & Marwan, 2016). Students of the climber type tend to have the desire to get better (Indra Kurniawan, Kusmayadi & Sujadi, 2015).

Someone with high AQ will be encouraged to get the best results by actively acting, always taking advantage of the opportunities that exist, and having the willingness to learn independently (Novilita & Suharnan, 2013). Yanti and Syazali suggest that the high and low AQ can be measured using an indicator which consists of four dimensions including Control, Origin, Reach and Endurance (Yanti & Syazali, 2013), as shown in Table 1. The AQ score can be counted using the formula $C + O_2 + R + E = AQ$ (Stoltz, 2000).

Table 1. The Indicators of Adversity Quotient

Indicators (AQ Dimension: CO_2RE)		Description
C	Control; the level of control toward the events lead to problems	Students' self-control when sensing a problem
O_2	Origin and Ownership	O_r : The ownership of the origin of problems O_w : The ownership toward the problems
R	Reach; how far the problem could reach other aspects of live	The students' ownership of how far the problem could reach other aspects of live
E	Endurance	Students' perception of how long will the problems going on

METHOD

This study uses the qualitative approach with descriptive research type. This research was conducted at the Mathematics Education Study Program. The research subjects were the students who took Real Analysis courses in the second semester of the 2017/2018

Academic Year with a total of 25 students. Sampling technique used was purposive. The data collecting techniques were documentation, tests, and interviews. The students first fill out a questionnaire of Adversity Quotient to later group them into three categories namely climbers, campers, and quitters. From the questionnaire, the AQ score was obtained.

Furthermore, the categorization of AQ in this study refers to the determination of the interval category (Azwar, 2002), based on the theoretical mean (μ) and standard deviation (σ). The categorization criteria can be seen in Table 2 below. Where X states, the AQ score obtained.

Table 2. Categorization of AQ

Criteria	Category
$\mu + 1{,}0\sigma \le X$	High
$\mu - 1{,}0\sigma \le X < \mu + 1{,}0\sigma$	Medium
$X < \mu - 1{,}0\sigma$	Low

After analysis of the AQ, questionnaire had been conducted, and the results were obtained as presented in Table 3.

Table 3. Results of Adversity Quotient Questionnaire

Dimension	Number of Subjects	Score				Mean		Standard Deviation	
		t-Min	t-Max	e-Min	e-Max	Theoretical	Empirical	Theoretical	Empirical
C	25	8	32	20	28	20	23.32	4	1.95
O_2	25	11	44	26	41	27.5	32.84	5.5	3.80
R	25	12	48	28	41	30	35.80	6	3.11
E	25	9	36	21	35	22.5	26.28	4.5	3.17
AQ	25	40	160	101	140	100	118.24	20	9.04

Furthermore, from the data in Table 3, the theoretical mean and standard deviations were then used to determine the AQ categorization criteria in this research. The categorization criteria are in Table 4.

Table 4. AQ Categorization

Criteria	Category
$120 \le X$	High
$80 \le X < 120$	Medium
$X < 80$	Low

For the category of the Adversity Quotient, the highest category is assumed to be the Climbers category, and the medium category is assumed to be the Campers category, while the lowest category is assumed to be the Quitters category. Then the students were given a test of mathematical proof, the results of the tests are analyzed as a determination for the next process, namely interviews. From each category selected the work results of students with the type of error that represents other students and then selected as a subject who will be confirmed the results of their work through interviews. As for the analysis of the results of interviews conducted by going through several stages, namely data reduction, data presentation, and final conclusion.

RESULT AND DISCUSSION

Based on the data obtained, grouping students is based on Adversity Quotient by referring to Table 5.

Table 5. Student Grouping Results

Category	Number of Students	Percentage
High	12	48%
Medium	13	52%
Low	0	0%

This study did not find any students with the quitter Adversity Quotient category. The result is taken from the campers and climbers category. The test questions given were three mathematical proof questions. The first problem is as follows:

Given the function $f: \mathbb{R} \to \mathbb{R}$ defined by

$$f(x) := \begin{cases} x & if\ x\ rational \\ 0 & if\ x\ irrational \end{cases}$$

Prove that $\lim_{x\to 0} f(x)$ exist, but $\lim_{x\to c} f(x)$ do not exist if $c \neq 0$.

The answer from the climber- type students can be seen in the following Figure 1.

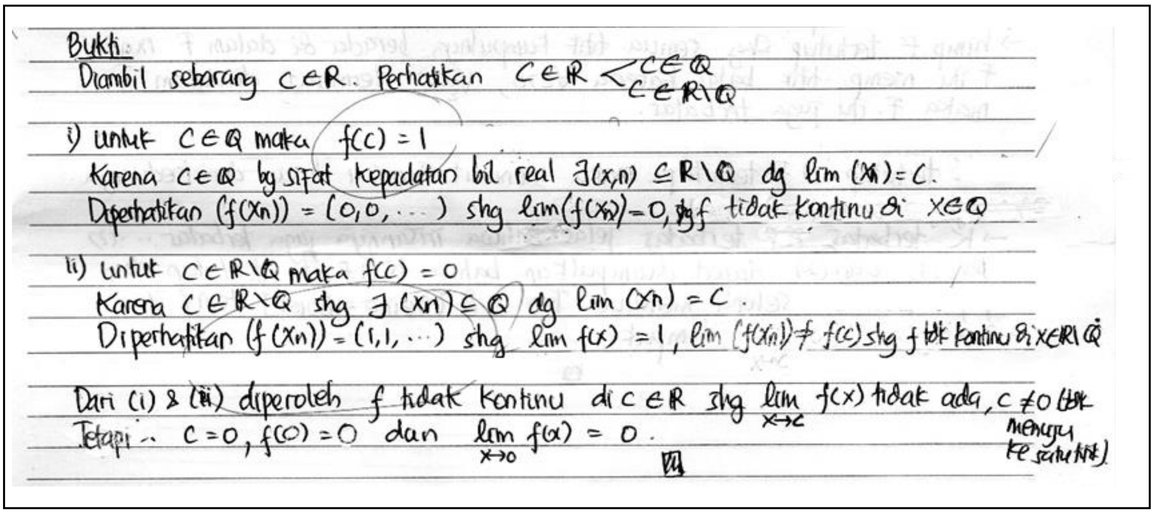

Bukti:
Diambil sebarang $c \in \mathbb{R}$. Perhatikan $c \in \mathbb{R} < \begin{matrix} c \in \mathbb{Q} \\ c \in \mathbb{R}\setminus\mathbb{Q} \end{matrix}$
i) untuk $c \in \mathbb{Q}$ maka $f(c) = 1$
Karena $c \in \mathbb{Q}$ bg sifat kepadatan bil real $\exists (x_n) \subseteq \mathbb{R}\setminus\mathbb{Q}$ dg $\lim (x_n) = c$.
Diperhatikan $(f(x_n)) = (0, 0, \ldots)$ shg $\lim (f(x_n)) = 0$, shg f tidak kontinu di $x \in \mathbb{Q}$
ii) untuk $c \in \mathbb{R}\setminus\mathbb{Q}$ maka $f(c) = 0$
Karena $c \in \mathbb{R}\setminus\mathbb{Q}$ shg $\exists (x_n) \subseteq \mathbb{Q}$ dg $\lim (x_n) = c$.
Diperhatikan $(f(x_n)) = (1, 1, \ldots)$ shg $\lim f(x) = 1$, $\lim (f(x_n)) \neq f(c)$ shg f tdk kontinu di $x \in \mathbb{R}\setminus\mathbb{Q}$
Dari (i) & (ii) diperoleh f tidak kontinu di $c \in \mathbb{R}$ shg $\lim_{x \to c} f(x)$ tidak ada, $c \neq 0$ (tdk menuju ke satu titik)
Tetapi: $c = 0$, $f(0) = 0$ dan $\lim_{x \to 0} f(x) = 0$. ∎

Figure 1. The Results of Climbers Type Students' Work for Question Number 1

Based on Figure 1, it appears that students proved that the function did not have a limit by connecting it to a continuous function. Students thought that if a function is not continuous, the function has no limit. This is a wrong understanding. Furthermore, when further analyzed was conducted, it appears that when $c \in \mathbb{Q}$ obtained $f(c) = 1$. Of course, this is not true based on the function definition given in the question. After these things were confirmed to the students concerned, it turns out that students were still referring to the example discussed in the previous lecture. In addition, students still had the wrong understanding regarding the limit of functions and continuous functions. Therefore, students experienced errors in determining strategies to solve these problems. While in the process, there were still a number of incorrect steps. The results of this analysis can be seen in the following Table 6.

Table 6. Results of Analysis of Type Student Work Climber for Number 1.

Types of Errors	Analysis Results
Reading Error	Students do not experience problems in reading errors. Students understand the problem given in question number 1.
Comprehension Error	Students know and understand what information given by question number 1 and what must be proven. It is seen that students are able to write the definition of functions given in mathematical language.
Transformation Error	Students make mistakes in this type. It is seen that the strategy used by students is to show that a continuous function has no limit. Of course, this is in contrast to the facts.
Process Skill Error	Students still make mistakes in carrying out some verification steps. It can be seen $c \in \mathbb{R}\setminus\mathbb{Q}$ is written $f(c) = 0$. course this is not in accordance with the definition given.
Encoding Error	Students have not been able

Types of Errors	Analysis Results
	to determine the resolution of this problem correctly.

Thus, it can be seen that students experience a tendency for transformation error and process skill error. Next in Figure 2, the results of Camper type students for question number 1. The results of this work indicated that there was a mismatch between the answers and the questions. The students were required to prove that the limit for $x \to 0$ exists, while the limit for $x \to c$ and $c \neq 0$ do not exist. However, it can be seen that students show f continuous in $x = 0$ and not continuous in $x \neq 0$. After being confirmed through interviews, it turns out that students were fixated on the sample questions that were discussed at the lecture. Students understood when they were asked to prove that the limit exists, but did not know what can be used from the information given by the question. So that students had difficulty in determining the next step for the verification process.

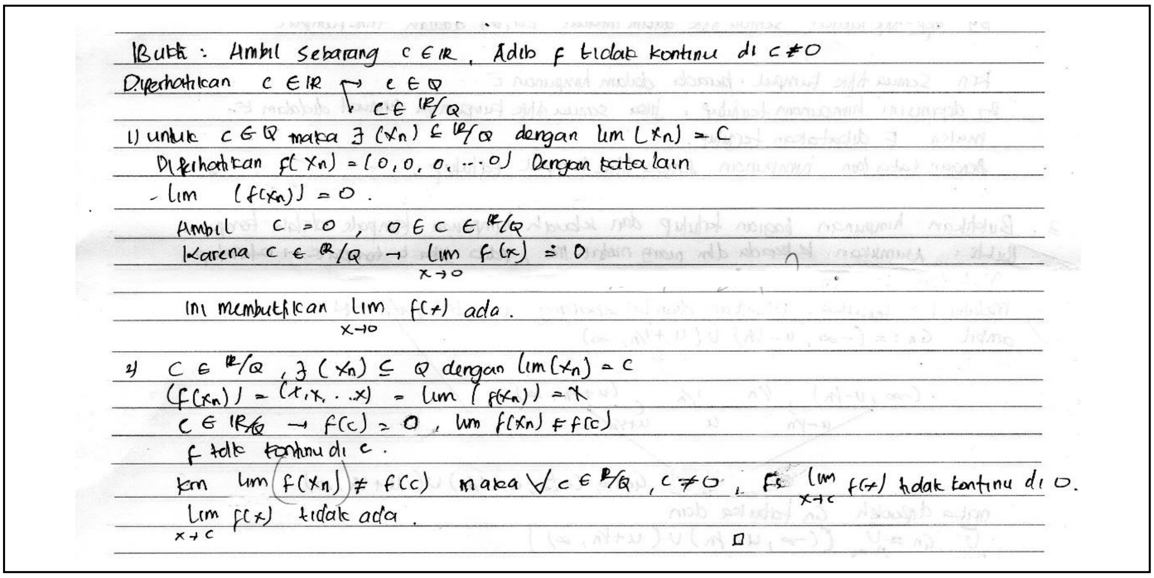


Figure 2. The Results of Camper-type Students' Answer for Question Number 1

The results of the analysis are in the following Table 7.

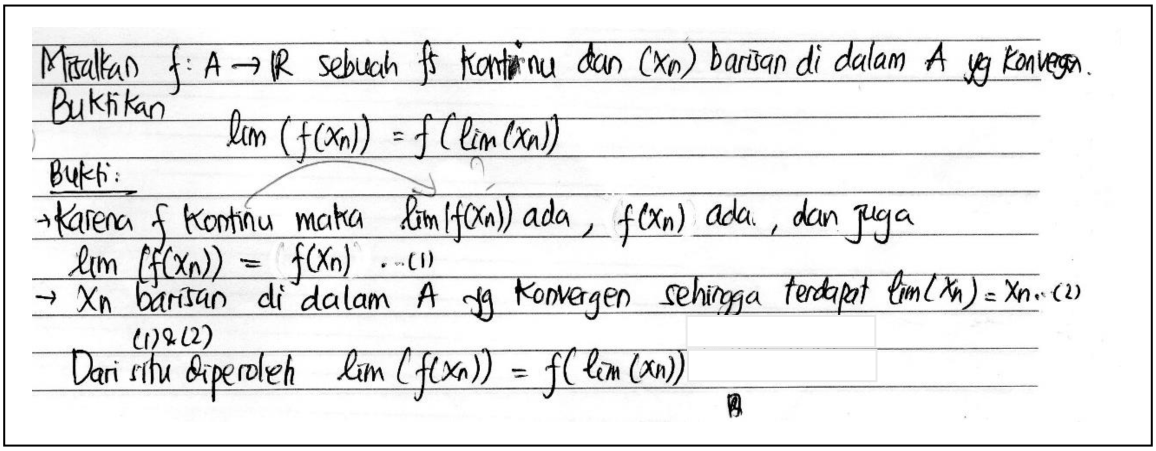
Table 7. The Analysis Results of Camper-type Students' Answer for Question Number 1

Type of Error	Analysis Results
Reading Error	If you see the results of student work above, it seems that there is an error in the reading process. Because there is a mismatch between questions and answers. However, after being confirmed through interviews, it turned out that students were aware of that. So, students know that the answer given is not by the question.
Comprehension Error	Students provide such answers because they only know a little from the information. The rest of the students did not know what could be used from the information provided by the question.
Transformation Error	Students did not know what strategies to use to solve problems in this question.
Process Skill Error	Students do not carry out verification procedures correctly.
Encoding Error	Students have not been able to determine the resolution of this problem correctly.

Thus, on the question, it can be seen that the student tend to do comprehension errors, transformation errors, and process error skills. For the second question, it is still in the form of mathematical proof. The second question is as follows:

For example, let $f: A \to \mathbb{R}$ be continuous on R and let x_n sequences in A is convergent. Prove the $\lim(f(x_n)) = f(\lim(x_n))$.

Following is the work results from the climber-type students for the second question.



Misalkan $f : A \to \mathbb{R}$ sebuah fs kontinu dan (X_n) barisan di dalam A yg konvergen.
Buktikan $\lim (f(X_n)) = f(\lim(X_n))$
Bukti:
→ Karena f kontinu maka $\lim(f(X_n))$ ada, $f(X_n)$ ada, dan juga
$\lim (f(X_n)) = f(X_n)$ ...(1)
→ X_n barisan di dalam A yg konvergen sehingga terdapat $\lim(X_n) = X_n$ ..(2)
(1)&(2)
Dari situ diperoleh $\lim(f(x_n)) = f(\lim(x_n))$
∎

Figure 3. The Results of Climber-type Students' work for question number 2

From Figure 3, it appears that students proved this case by connecting the concepts of continuous functions and limit lines. In the first stage, students took advantage of the concept that when a function is continuous, the limit is exist, the function value is exist, and the limit value is the same as the function value. However, students did not provide a justification regarding the line of (x_n) used. Next, students used the concept of converging sequence for the next process. However, it appears that students wrote down $\lim(x_n) = x_n$ since (x_n) was a convergent sequence. Then the conclusion was that the limit value was equal to the value of its function. After being confirmed to the students concerned, information was obtained that the students used line (x_n) on continuous concepts so that they could be linked to the information given, namely sequences (x_n) convergent. Furthermore, when the students wrote $\lim(x_n) = x_n$ in hopes that they could be connected to the concept of continuous function. From the results of this confirmation, it can be seen that the students used the correct strategy, but at the time of execution, it seems that students used inappropriate methods. Thus, it can be seen that in this problem students did not make a transformation error, but a process skill error. Furthermore, students had led to solving the problem, but the form of the answer given was still incorrect. It can be concluded that this thing is included in encoding errors. The results are presented in Table 8.

Table 8. The Results of analysis of the Climber-type Students' Work for Question Number 2

Type of Error	Analysis Results
Reading Error	Students do not experience problems related to reading.
Comprehension Error	Students understand the purpose of the problem, and it seems that students use all the information provided by the problem.
Transformation Error	Students have had a strategic idea to prove this case, namely by connecting the limit of the line and the continuous function. This is done by utilizing the properties that apply to the limit of functions and

Type of Error	Analysis Results
	continuous functions.
Process Skill Error	In the process, it appears that students are still writing inappropriate procedures. This can be seen from the statement $\lim(x_n) = x_n$. Of course, this statement raises questions, but there is no justification for this statement.
Encoding Error	Student answers have led to solving the problem, but the form of the answer given is not correct. Because there are some steps that are not clear and there is no justification.

Furthermore, the following (Figure 4) is the work result of the camper-type students for question number 2. After further observing the results of student work in Figure 4, students intend to prove this case by using formal definitions of continuous functions and limit functions. However, students do not provide a definition of continuous functions but a definition of limit functions. It seems that students have not been able to correctly identify what is informed by the problem and what can be utilized from the question information. It seems that students experience error comprehension.

Furthermore, in the process, the definition of convergent sequence does not appear in the results of students' work. There appears to be a statement $\forall\varepsilon > 0, \exists\delta > 0 \ni |x_n - \lim(x_n)| < \delta \rightarrow |f(x_n) - f(\lim(x_n))| < \varepsilon$ caused by the convergence of lines (x_n), but there should be an explanation before writing the statement above because if so, the causal relationship above is not suitable.

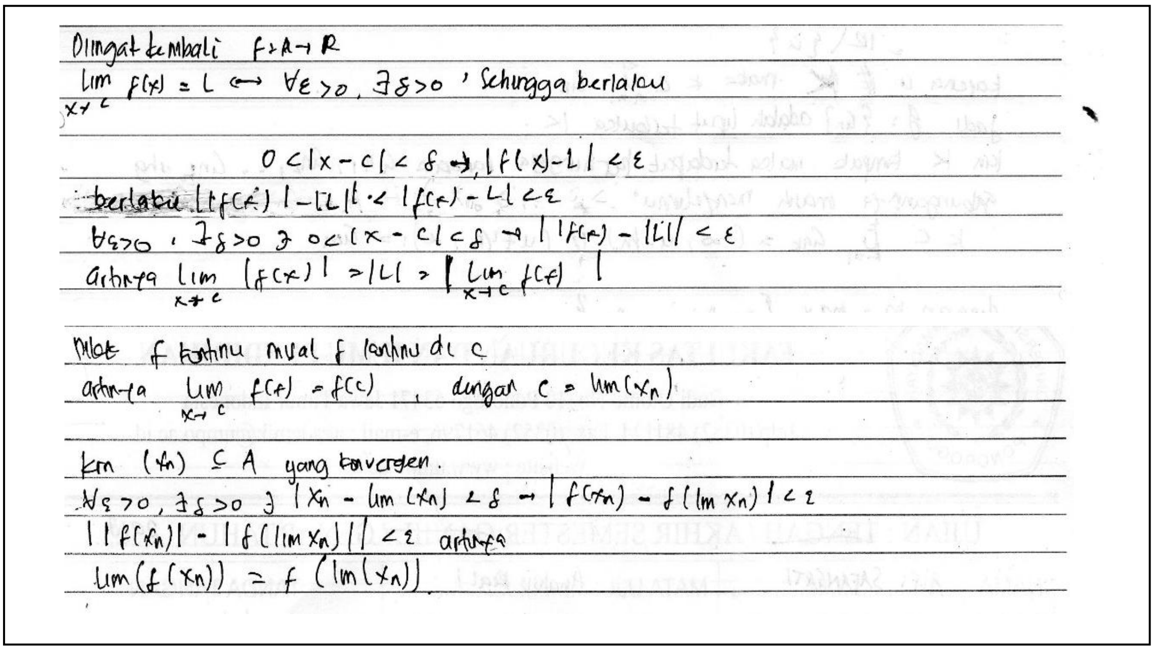


Figure 4. Results of Camper-type Students' Work for Question Number 2

After being confirmed with the students concerned, it turns out that students are still confused about how to use the concept of converging sequence. Therefore, students direct the answer to statement 1. It appears that in this problem, Camper-type students are similar to Climber-type students in the sense that they have the right problem-solving strategies, but made mistakes in carrying out the strategy.

Table 9. The Results of the Analysis of the Camper-type Students' Work for Question Number 2

Type of Error	Analysis Results
Reading Error	Students do not experience problems related to reading.
Comprehension Error	Students cannot use the information provided by the problem. The actual concept that needs to be reviewed in this problem is the definition of continuous functions. However, students instead provide a definition of limit functions.
Transformation Error	Students have the right strategy, namely by utilizing formal definitions. Furthermore, students try to associate the concept of continuous function with a line limit.
Process Skill Error	In the process, the student only mentions once the line limit is locked and there is no justification at all regarding the line limit.
Encoding Error	The results of the students' work have led to the completion of the desired final form. However, the verification procedure provided is still incorrect.

Thus, for question number 2, the Camper-type students have a tendency toward the comprehension errors, process skills error, and encoding errors. The third question is still in the form of mathematical proof. The third question is as follows:

Prove that the set of limit points of a set is closed.

The following is the result of students' work for the third question.

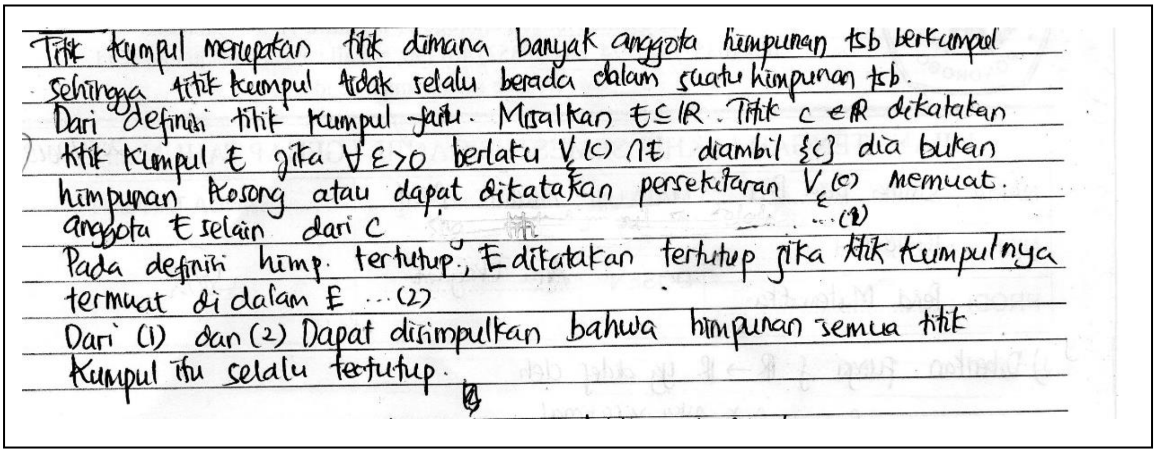

Titik kumpul merupakan titik dimana banyak anggota himpunan tsb berkumpul sehingga titik kumpul tidak selalu berada dalam suatu himpunan tsb.
Dari definisi titik kumpul yaitu. Misalkan $E \subseteq \mathbb{R}$. Titik $c \in \mathbb{R}$ dikatakan titik kumpul E jika $\forall \varepsilon > 0$ berlaku $V(c) \cap E$ diambil $\{c\}$ dia bukan himpunan kosong atau dapat dikatakan persekitaran $V_\varepsilon(c)$ memuat anggota E selain dari c …(1)
Pada definisi himp. tertutup, E dikatakan tertutup jika titik kumpulnya termuat di dalam E …(2)
Dari (1) dan (2) Dapat disimpulkan bahwa himpunan semua titik kumpul itu selalu tertutup.

Figure 5. Results of Climber-type Students' Work for Question Number 3

The result of the student work above shows that the strategy used to prove the case was the definition of a closed set. However, the reason or explanation given was not so strong to conclude. After being confirmed to the students, they were still confused to provide mathematical proof of justification. As a result, students provided compelling conclusions. In this case, it can be seen that the students already had mathematical proof of ideas or strategies, but the same as in solving the previous questions, they still had problems with the execution of the strategy.

Table 10. Results of Analysis of the Climber-type Students' Work for Question Number 3

Types of Errors	Analysis Results
Reading Error	Similar to other cases, students do not experience problems related to reading.
Comprehension Error	Students have understood information that can be used from the questions given. It is seen that

Types of Errors	Analysis Results
	students can provide definitions of gathering points and definitions of closed sets.
Transformation Error	Students have the right strategy, namely by using the definition of closed set.
Process Skill Error	When executing an existing strategy, students are still lacking in giving justification at each step.
Encoding Error	The results of this students' work have led to completion and have the desired final form. But the justification is still lacking, and it can be said that the justification is still not strong enough to conclude this proof.

Thus, it can be seen that the climber-type students have a tendency to make mistakes in the process skills error and encoding errors (Indra Kurniawan, Kusmayadi & Sujadi, 2015). The following is the work of the camper-type students for question number 3.

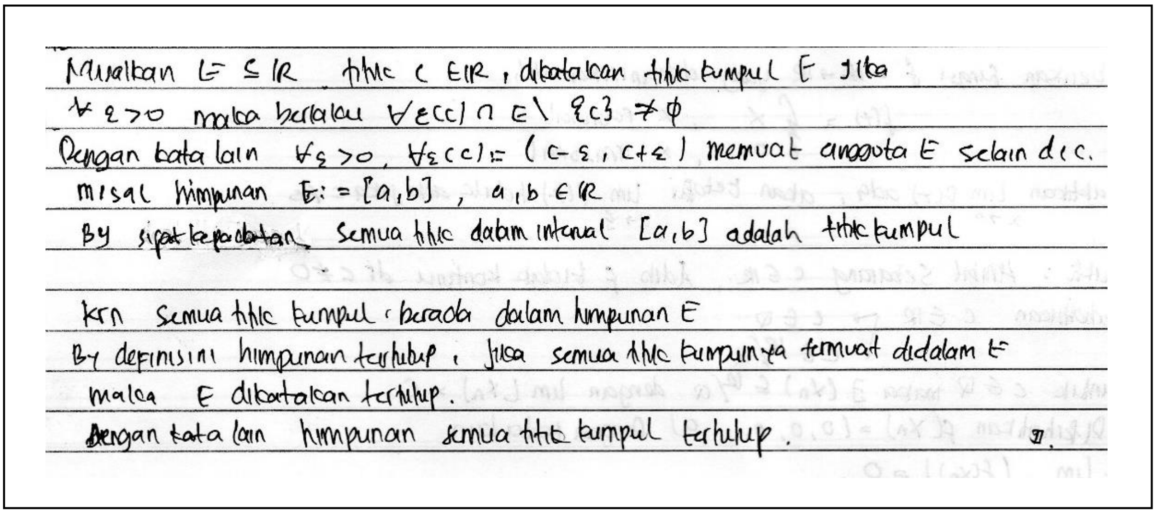

Misalkan $E \subseteq \mathbb{R}$ titik $c \in \mathbb{R}$, dikatakan titik kumpul E jika
$\forall \varepsilon > 0$ maka berlaku $V_\varepsilon(c) \cap E \setminus \{c\} \neq \phi$
Dengan kata lain $\forall \varepsilon > 0$, $V_\varepsilon(c) := (c-\varepsilon, c+\varepsilon)$ memuat anggota E selain dari c.
misal himpunan $E := [a,b]$, $a, b \in \mathbb{R}$
By sifat kepadatan, semua titik dalam interval $[a,b]$ adalah titik kumpul
krn semua titik kumpul berada dalam himpunan E
By definisi himpunan tertutup, jika semua titik kumpulnya termuat didalam E
maka E dikatakan tertutup.
Dengan kata lain himpunan semua titik kumpul tertutup. ∎

Figure 6. The results of the Camper-type Students' Work for Question Number 3

The result of question number 3 is generally similar to the answer of the climber-type students. The students use the definition of closed sets to prove this case. However, students encountered problems in how to justify this verification. So, students don't have problems in determining strategies, and problems arise when executing the strategy. This shows that the error that tends to occur for question number 3 is the process skill error and encoding error.

CONCLUSION

There are several types of errors that students tend to do in solving mathematical cases in the form of mathematical proof. For climber-type students, some types of errors that they tend to do in doing mathematical proof are transformation errors, process skill errors, and encoding errors. The camper-type students tend to do comprehension errors, transformation errors, process skills error, and encoding errors. In comprehension error, it can be seen that in compiling proof, the students understand the intention of the problem but do not know what information can be taken. For transformation error, it can be seen from the misunderstanding between the concept of continuous functions and limit functions. As for the process skill error, it can be seen from students' errors in writing mathematical proof, and there are still steps that are not accompanied by justification, or even steps that are not written correctly. The encoding error can be seen from the evidentiary steps that have been written down, have not been compiled with the correct flow, and there is still a lack of mathematical proof justification for drawing conclusions.

REFERENCES

Ah, N. I. (2016). Pengembangan Bahan Ajar Mata Kuliah Analisis Real Berbasis Pembuktian Pada Semester V UNMUH JEMBER. *Jurnal Gammath*, *I Nomor 2*, 26–32.

Azwar, S. (2002). *Penyusun Skala Psikologi*. Yogyakarta: Pustaka Pelajar Offset.

Bagus Nur Iman, Toto Nusantara, D. (2016). Diagnosis Kesalahan Siswa Kelas XI SMA Negeri Candipuro pada Materi Penerapan Turunan Fungsi. In *Seminar Nasional Matematika dan Pembelajarannya* (pp. 146–153). Malang: Universitas Negeri Malang.

Ekayanti, A. (2017). Diagnosis Kesalahan Mahasiswa Dalam Proses Pembuktian Berdasarkan Newmann Error Analysis. *Mosharafa: Jurnal Pendidikan Matematika*, *6*(1), 105–116.

Guntur Suhandoyo, P. W. (2016). Profil Kemampuan Berpikir Kreatif Siswa dalam Menyelesaikan Soal Higher Order Thinking Ditinjau dari Adversity Quotient (AQ). *MathEdunesa: Jurnal Ilmiah Pendidikan Matematika*, *3*(5), 156–165.

Hernadi, J. (2016). *Fondasi Matematika Dasar Berfikir Logis dan Kritis dalam Matematika*. Ponorogo: UMPO Press.

Hidayat, W. (2017). Adversity Quotient dan Penalaran Kreatif Matematis Siswa SMA dalam Pembelajaran Argument Driven. *KALAMATIKA*, *2*(1), 15–28.

Hidayat, W., Herdiman, I., Aripin, U., Yuliani, A., & Maya, R. (2018). Adversity Quotient (AQ) dan Penalaran Kreatif Matematis Mahasiswa Calon Guru. *Jurnal Elemen*, *4*(2), 230–242. https://doi.org/10.29408/jel.v4i2.701

Indra Kurniawan, Kusmayadi, T. A., & Sujadi, I. (2015). Proses Berpikir Kreatif Siswa Climber dalam Pemecahan Masalah pada Materi Peluang (Penelitian Pada Siswa Kelas XI IPA Semester II SMA Negeri I Polanharjo Tahun Ajaran 2014 / 2015), *3*(6), 599–612.

Ko, Y. Y., & Knuth, E. (2009). Undergraduate mathematics majors' writing performance producing proofs and counterexamples about continuous functions. *Journal of Mathematical Behavior*, *28*(1), 68–77. https://doi.org/10.1016/j.jmathb.2009.04.005

Leonard, L. (2017). Pengaruh Adversity Quotient (AQ) dan Kemampuan Berpikir Kritis Terhadap Prestasi Belajar Matematika, *28*(April 2014).

Novilita, & Suharnan. (2013). Konsep Diri Adversity Quotient dan Kemandirian Belajar Siswa. *Jurnal Psikologi*, *8*(1), 619–632.

Nurhayati. (2015). Pengaruh Adversity Quotient dan Motivasi Berprestasi Terhadap Prestasi Belajar Matematika. *Jurnal Formatif*, *3*(1), 72–77.

Rukmana, I., & Paloloang, B. (2016). Hubungan Adversity Quotient dengan Hasil Belajar Matematika Siswa Kelas XI SMA Negeri Model Terpadu Madani Palu, *3*(3), 325–333.

Sari, C. K., Sutopo, S., & Aryuna, D. R. (2016). The Profile of Students' Thinking in Solving Mathematics Problems Based on Adversity Quotient. *Journal of Research and Advances in Mathematics Education*, *1*, 36–48.

Sari, C. K., Waluyo, M., Ainur, C. M., & Darmaningsih, E. N. (2018). Logical errors on proving theorem. *Journal of Physics: Conference Series*, *948*(1). https://doi.org/10.1088/1742-6596/948/1/012059

Sari, D. P., Darhim, D., & Rosjanuardi, R. (2018). Errors of Students Learning with React Strategy in Solving The Problems of Mathematical. *Journal on Mathematics Education*, *9*(1), 121–128.

Stavrou, S. G. (2014). Common Error and Misconceptions in Mathematical Proving by Education Undergraduates. *Issues in the Undergraduate Mathematics Preparation of School Teachers*, *1*(March), 1–8.

Stefanowich, A. (2014). Proofs and Mathematical Reasoning. University of Birmingham.

Stoltz. (2000). *Mengubah Hambatan Menjadi Peluang*. Jakarta: Grasindo.

Sucipto, L., & Mauliddin. (2016). Analisis Kesulitan Mahasiswa dalam Memahami Konsep Bilangan Real. *Beta Jurnal Tadris Matematika*, *9*(2).

Sukoriyanto, S., Nusantara, T., Subanji, S., & Chandra, T. D. (2016). Students' Errors in Solving the Permutation and Combination Problems Based on Problem Solving Steps of Polya. *International Education Studies*, *9*(2), 11. https://doi.org/10.5539/ies.v9n2p11

Wardiana, W. dan S. Z. (2014). Hubungan Antara Adversity Quotient (AQ) dan Minat Belajar dengan Prestasi Belajar Matematika pada Siswa Kelas V SD di Kelurahan Pedugan. *Jurnal Mimbar PGSD Universitas Pendidikan Ganesha*, *2*(1).

Wijaya, A., Heuvel-panhuizen, M. Van Den, Doorman, M., & Robitzch, A. (2014). Difficulties in solving context-based PISA mathematics tasks : An analysis of students ' errors. *The Mathematics Enthusiast*, *11*(3), 555–584. https://doi.org/10.1139/t02-118

Yani, M., Ikhsan, M., & Marwan, M. (2016). Proses Berpikir Siswa Sekolah Menengah Pertama dalam Memecahkan Masalah Matematika Berdasarkan Langkah-langkah Polya. *Jurnal Pendidikan Matematika*, *10*(1), 43–58.

Yanti, A. P., Koestoro, B., & Sutiarso, S. (2018). The Students' Creative Thinking Process based on Wallas Theory in Solving Mathematical Problems viewed from Adversity Quotient /Type Climbers. *Al Jabar: Jurnal Pendidikan Matematika*, *9*(1), 51–61.

Yanti, A. P., & Syazali, M. (2013). Analisis Proses Berpikir Siswa dalam Memecahkan Masalah Matematika Berdasarkan Langkah-Langkah Bransford dan Stein Ditinjau dari Adversity Quotient. *Al-Jabar: Jurnal Pendidikan Matematika*, *7*(3), 64.